

M.Sc. (Maths) Semester - 2 (CBCS) Examination

May/June -2021 [NEW COURSE]

METHODS IN PARTIAL DIFFERENTIAL EQUATIONS – 2 (CORE)

Time: 1:30 Hours

Marks: 42

Instructions:

1. Figure to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any three of the following questions.

Q.1(A) Verify that the equation $z(z+y^2)dx + z(z+x^2)dy - xy(x+y)dz = 0$ is integrable and find its primitive. [04]

Q.1(B) Attempt any two [10]

1. Find the integral curves of the equations $\frac{dx}{x+z} = \frac{dy}{y} = \frac{dz}{z+y^2}$.
2. Find the orthogonal trajectories on the cone $x^2 + y^2 = z^2 \tan^2 \alpha$ of its intersections, with the family of planes parallel to $z = 0$.
3. Verify that the equation $yz(y+z)dx + xz(x+z)dy + xy(x+y)dz = 0$ is integrable and find its solution.

Q.2(A) Eliminate the arbitrary function f from the equation $f(x^2 + y^2 + z^2, z^2 - 2xy) = 0$. [04]

Q.2(B) Attempt any two [10]

1. Find the integral surface of the linear partial differential equation $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$.
2. Find the surface which intersects the surfaces of the system $z(x+y) = c(3z+1)$ orthogonally passes through the circle $x^2 + y^2 = 1, z = 1$.
3. Show that the general solution of the linear partial differential equation $Pp + Qq = R$ is $F(u, v) = 0$, where F is arbitrary function and $u(x, y, z) = c_1$ and $v(x, y, z) = c_2$.

Q.3(A) Find the general solution of $p^2 - q^2 = x - y$ using Charpit's method. [04]

Q.3(B) Attempt any two [10]

1. Find the general solution of $(p+q)(px+qy) = 1$ using Charpit's method.
2. Find the general solution of $p^2z^2 + q^2 = 1$ using Jacobi's method.
3. Find the general solution of $p^2x + q^2y = z$ using Jacobi's method.

Q.4(A) Classify the following differential equation into hyperbolic/parabolic/elliptic. [04]

(I) $r = t$ (II) $r + x^2t = 0$ (III) $yr + (x+y)s + xt = 0$ (IV) $y^2r + t = 0$

Q.4(B) Attempt any two [10]

1. Convert $x^2r + 2xys + y^2t = 0$ into canonical form.
2. Find the general solution of the differential equation $Z_{xx} - Z_{yy} = x - y$.
3. Show that $F(D, D')(e^{ax+by}\phi(x, y)) = e^{ax+by}F(D+a, D'+b)\phi(x, y)$.

Q.5(A) Verify that the equation $yzdx + xzdy + xydz = 0$ is integrable and find its primitive. [04]

Q.5(B) Attempt any two [10]

1. Find the general integral of the linear partial differential equation $y^2p - xyq = x(z-2y)$.
2. Find the general solution of $z^2 = pqxy$ using Jacobi's method.
3. Convert $r + x^2t = 0$ into canonical form.
